Charge Symmetry Violation Effects in Pion Scattering off Light Nuclei

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Abstract

We discuss the theoretical and experimental situation of charge symmetry violation (CSV) effects in scattering of π^+ and π^- on deuterium (D) and $^3He/^3H$. Accurate comparison of data for both types of targets provides evidence for the presence of CSV effects. While there are indications of the CSV effect in deuterium, it looks much more pronounced in the case of $^3He/^3H$. We provide a description of the CSV effect in terms of single- and double-scattering amplitudes. The Δ-mass splitting is taken into account. Theoretical predictions are compared with existing experimental data.

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I. INTRODUCTION

The study of CSV in the interaction of pions with nuclei in the Delta resonance region has been of considerable interest for the last two decades. The interaction of pions with light nuclei such as ^2H [1] $-$ [9], $^3He/^3H$ [9] $-$ [13], and ^4He [14] has attracted particular attention. However, we note that quite a large data set also exists for scattering of π^+ and π^- on ^{12}C, ^{16}O, and ^{40}Ca as well [15].

From the point of view of theory, the advantage of searching for CSV in the scattering of pions from light nuclei is that one can describe pion scattering in these systems in a relatively straight-forward manner. With this in mind, we limit ourselves to the consideration of the scattering of pions from deuterium, ^3He, and ^3H. Moreover, we anticipate that CSV effects are considerably diminished in the case of pion scattering from heavier nuclei because of the importance of processes such as absorption.

First, in order to evaluate the scale of CSV effect, we focus our theoretical efforts primarily on πd scattering. In a following article, we will develop the formalism further to investigate CSV in the three-nucleon system.

A detailed analysis of the experimental situation will be given in the next section. Here, we want only to point out that in order to make a comparison between experimental data related to different projectile or target, we must deal with the same experimental measurables. Historically, the CSV experimental data were given in terms of asymmetry, A_π for the deuteron:

$$A_\pi = \frac{d\sigma/d\Omega(\pi^- d) - d\sigma/d\Omega(\pi^+ d)}{d\sigma/d\Omega(\pi^- d) + d\sigma/d\Omega(\pi^+ d)}, \tag{1}$$

and in terms of ratios r_1 and r_2, and superratio R for the $^3He/^3H$ case:

$$\begin{aligned} r_1 &= \frac{d\sigma/d\Omega(\pi^{+3}H)}{d\sigma/d\Omega(\pi^{-3}He)}, \\ r_2 &= \frac{d\sigma/d\Omega(\pi^{-3}H)}{d\sigma/d\Omega(\pi^{+3}He)}, \\ R &= r_1 r_2. \end{aligned} \tag{2}$$

Both interactions $\pi^{+3}H$ and $\pi^{-3}He$ for the ratio r_1, and $\pi^{-3}H$ and $\pi^{+3}He$ for the ratio r_2 are isomirror interactions. Therefore, if charge symmetry is strictly observed, both r_1 and r_2 would be equal to 1.0. Of course, the Coulomb interaction is not charge symmetric and would have to be taken into account. The superratio R is the product r_1 and r_2. So, if charge symmetry is universally true, R is also equal to 1.0.

The experimental data suggests evidence for a small effect in A_π for the deuteron (e. g. $A_\pi \simeq 2\%$ at 143 MeV [3]) with some indication of structure at scattering angles around 90° in cm frame. At the same time, a sizable effect is clearly seen in the $^3He/^3H$ case. For example, $r_2 = 0.7 \pm 0.1$ for $T_\pi = 256$ MeV and $\theta = 82°$ [12]. Theoretical predictions for the asymmetry A_π in the deuterium case were given in Ref. [3]. To describe the asymmetry, authors of Ref. [3] used a single-scattering approximation with allowance for differently charged Δ's(1232). In this approximation, the CSV effect proved to be independent of the scattering angle with typical value proportional to $\delta m_\Delta/\Gamma_\Delta$. Approximately the same approach was used in the $^3He/^3H$ case in the paper [9].

A different approach for the $^3He/^3H$ case was suggested in the paper [16]. Authors of this paper used an optical potential to describe the pionic $^3He/^3H$-amplitudes. The radial dependence of πA potentials was determined in terms of matter and spin densities for ^3He and ^3H. The Coulomb-nuclei interference effect in the vicinity of minima in differential cross sections was reported as the main reason for the CSV effect in [16] approach. However, this interpretation was disputed by Briscoe and Silverman [17] because the authors of [16] obtained structure only near the 90° in r_2 but could not at all explain the overall behavior of the experimental data.

In our investigation, we shall study the role of double-scattering on CSV. It is widely known that the single-scattering approximation reproduces a differential cross section fairly well in the forward hemispere. But for scattering angles beyond 90°, the double-scattering term is important and should be included [18].

In Section III, we explain how the basic ingredients of the scattering amplitude and constraints such as single- and double-scattering, and Coulomb interaction are combined. These results and the prospect for improvements are summarized in Section IV.

II. ANALYSIS OF EXPERIMENTAL SITUATION

The CSV effect was first observed in the difference of total $\pi^{\pm}d$ cross sections in PSI and reported in [1]. This has been widely discussed, see, e. g. the book by Ericson and Weise [19]. There have been several measurements for both π^+d and π^-d. The first systematic study of the CSV effect in the differential $\pi^{\pm}d$ cross sections was done at LAMPF and presented in the paper [20]. Soon after, the asymmetry A_π for $T_\pi = 143$ MeV was presented for the range of laboratory scattering angles between 20° and 115° [3]. The experiment was repeated for approximately the same range of scattering angles at $T_\pi = 256$ MeV [4]. We note that the structure in the asymmetry seen in [3] was not seen in the TRIUMF measurements of [5]. Meantime, some indications for the CSV effects were also obtained at low energies 30, 50, and 65 MeV at TRIUMF [6,7]. We also mention the high-energy Gatchina data at $T_\pi = 417$ MeV [8], which also shows some indications on CSV.

We recall that the asymmetry (1), and ratios (2), are the two different measures of CSV-effects. As in the $^3He/^3H$-case, we denote the ratio $r = r_1 = r_2$

$$r = \frac{d\sigma/d\Omega(\pi^-d)}{d\sigma/d\Omega(\pi^+d)} = 1 + \epsilon.$$

Then, in the case of small magnitudes of CSV, we get

$$A_\pi \approx \epsilon/2.$$

Clearly, the tiny effect requires high quality data.

Smith *et al.* [5] reported a -1.5% asymmetry in the πd cross sections at back angles, with uncertainties of 0.6% at the different angles. As far as we are aware, there are no πd measurements at an accuracy better than $\delta A_\pi = 0.8\%$, which is approximately the size of the effect that we calculate. Another way to demonstrate the smallness of the effect is via a partial-wave analysis (PWA) of the πd scattering data. This is shown in (Fig. 1).

III. THEORETICAL CONSIDERATION OF CSV-EFFECT IN DEUTERON

We see two possible ways to interpret the experimental situation:

- The *first* way is in the following. One may conclude that there is really no effect in deuterium in accordance with statement [5] and the effect in the $^3He/^3H$ case is influenced correspondingly by specific three-body configurations of ^3He and ^3H. By this, we mean the possible influence of the three-body CSV forces which are absent in ^2H case and/or differences in description of ^3He and ^3H wave functions as a consequence of an additional Coulomb repulsion between two protons in the ^3He case.
- The *second* scenario is to suggest that the effect may be seen in both cases ^2H and $^3He/^3H$. But in the deuterium, the effect is small in comparison with the $^3He/^3H$. There should still be some angular dependence for the CSV effect in the deuterium. However, Masterson *et al.* [3] have shown that within the impulse single-scattering approximation the angular dependence for CSV is absent when only scattering via the P_{33} is considered. The inclusion of others S- and P-waves does not change the situation dramatically as all the phases except P_{33} are small in the region of interest. So, we need to look beyond the single-scattering approximation and to consider multiple scattering of pions.

1. Single-Scattering Approximation

   Everywhere below, we shall use the following notations: $k_{cm} = \frac{m}{m+\omega}k$, $w = m + \omega - \frac{k^2}{2(m+\omega)}$, where ω is the pion energy, w_i are the masses of isobars, and here and below latin indices $1-4$ in the notations of amplitudes, masses and widths mean the corresponding isobar isospin state:

$$i = 1,\ 2,\ 3,\ 4$$

   for

$$\Delta^{++},\ \Delta^{+},\ \Delta^{0},\ \Delta^{-}.$$

   We also will consider mean values $w_0 = 1232\ MeV$ and suppose $\Gamma_{el} = \Gamma_{tot} = \Gamma_0 = 120\ MeV$. The values w_i $(i = 1,\ 2,\ 3,\ 4)$, we calculate according to the formula from the book [19] (page 124, Eq. (4.16)):

$$w_i = a - b\ I_i + c\ I_i^2,$$

   where I_i is the 3-d component of isospin for the $i - term$ from the Δ-multiplet. Using the average resonance values from the PDG [22], we get $a = 1231.8\ MeV$, $b = 1.38\ MeV$, and $c = 0.13\ MeV$.

   In this approximation, the πd amplitude is the sum of the two Feynman diagrams shown in Fig. 2.

   The elementary πN amplitude in terms of $\delta_{33}(k)$ phase looks like the following:

$$\hat{f}_{\pi N} = \frac{1}{2ik}(e^{2i\delta_{33}(k)} - 1)\frac{2\ (1 + \vec{t}\cdot\vec{\tau})}{3}(2\ \hat{\vec{k}}\cdot\hat{\vec{k\prime}} + i\ \vec{\sigma}\cdot[\hat{\vec{k}}\times\vec{k\prime}]) \tag{3}$$

and is the operator in spin and isospin space of the πN system. The deuteron wave function in S-wave approximation is $\frac{1}{\sqrt{2}}\psi_d(p) w_2^+(\vec{\epsilon}\cdot\vec{\sigma})\sigma_2 w_1^*$ (here w_1 and w_2 are the nucleon spinors and $\vec{\epsilon}$ is the polarization vector of deuteron), and the expression for amplitude f_1, which correspond to the diagram Fig. 2a, has the form:

$$f^{(1)}_{\pi d} = \frac{2}{E^{\pi d}_{cm}} \int \frac{d\vec{p}}{(2\pi)^3} E^{\pi N}_{cm} f_{33}(k_{cm}) \psi_d(\vec{p}) \psi_d(\vec{p} - \frac{\vec{\Delta}}{2}) \left(2(\vec{\epsilon}\cdot\vec{\epsilon\prime})(\hat{\vec{k}}\cdot\hat{\vec{k\prime}}) - [\vec{\epsilon}\times\vec{\epsilon\prime}]\cdot[\hat{\vec{k}}\times\hat{\vec{k\prime}}] \right). \quad (4)$$

Here $\vec{\Delta} = \vec{k} - \vec{k\prime}$ is the 3-dimension momentum transfer; $f_{33}(k) = \frac{1}{2ik}(e^{2i\delta_{33}(k)} - 1)$; $\vec{\epsilon}(\vec{\epsilon\prime})$ is the polarization vector of initial (final) deuteron; $\hat{\vec{k}} = \vec{k}_{cm}/k_{cm}$ and $\hat{\vec{k\prime}} = \vec{k\prime}_{cm}/k_{cm}$ are the units vectors, where $\vec{k}_{cm}(\vec{k\prime}_{cm})$ is the momentum of initial (final) pion in the rest frame of subprocess $\pi N \to \pi N$.

At this stage, we make some simplifications. We shall neglect Fermi motion of the nucleon and consider (for a while) the expression (4) in the static limit, i. e. $\omega/m \to 0$. Then, $2E^{\pi N}_{cm}/E^{\pi d}_{cm} \to 1$, $k_{cm} \to k$. So, we get

$$\hat{f}^{(1)}_{\pi d} = \frac{4}{3} f_{33}(k) \left(2(\vec{\epsilon}\cdot\vec{\epsilon\prime})(\hat{\vec{k}}\cdot\hat{\vec{k\prime}}) - [\vec{\epsilon}\times\vec{\epsilon\prime}]\cdot[\hat{\vec{k}}\times\hat{\vec{k\prime}}] \right) \int \Psi_D^2(r) e^{i\frac{\vec{\Delta}}{2}\cdot\vec{r}} d\vec{r}. \quad (5)$$

For this amplitude, the differential cross section with the unpolarized initial deuteron has the following form

$$\frac{d\sigma^{(1)}_{\pi d}}{d\Omega} = \frac{32}{27}(6\cos^2\theta + \sin^2\theta)|f_{33}(k)|^2 F_D^2(\Delta), \quad (6)$$

where $F_D(\Delta) = \int \Psi_D^2(r) e^{\frac{i\vec{\Delta}\cdot\vec{r}}{2}} d\vec{r}$. This expression agrees with that given in Ref. [3]. The ratio 6:1 between the terms proportional to $\cos^2\theta$ and $\sin^2\theta$ reflects the ratio of non-spin-flip to spin-flip amplitudes in this approximation.

2. Charge Symmetry Breaking Effect

First consider the elementary $\pi^+ p$ amplitude in terms of a $\Delta(1232)$ pole. The amplitude looks like a standard Breit-Wigner amplitude

$$f_{\pi^+ p} = -\frac{1}{2k} \frac{\Gamma_1}{w - w_1 + i\,\Gamma_1/2}, \quad (7)$$

where w_1 and Γ_1 is the mass and the full width, respectively, of the Δ^{++} resonance. Making linear expansion of this amplitude around the mean value of the mass w_0 and the width Γ_0 for the Δ resonance, we get

$$f_{\pi^+ p} = -\frac{1}{2k} \frac{\Gamma_0}{w - w_0 + i\,\Gamma_0/2}\left(1 + \frac{\delta\Gamma_1}{\Gamma_0} + \frac{\delta w_1 - i\,\delta\Gamma_1/2}{w - w_0 + i\,\Gamma_0/2}\right), \quad (8)$$

where $\delta\Gamma_1 = \Gamma_1 - \Gamma_0$ and $\delta w_1 = w_1 - w_0$. So, using Eq. (8), we get that the charge asymmetry in $\pi^\pm d$ scattering in this approximation is

$$A_\pi = \frac{3}{4}\frac{C_\Gamma(w-w_0)^2+(w-w_0)C_M\Gamma_0}{\Gamma_0[(w-w_0)^2+\Gamma_0^2/4]}, \tag{9}$$

where the parameters C_M and C_Γ are expressed in terms of Δ mass and width splitting:

$$C_M = \delta w_4 + \frac{1}{3}\delta w_3 - \frac{1}{3}\delta m_2 - \delta m_1 \simeq 4.6\ MeV,$$
$$C_\Gamma = \delta\Gamma_4 + \frac{1}{3}\delta\Gamma_3 - \frac{1}{3}\delta\Gamma_2 - \delta\Gamma_1 \simeq 1.7\ MeV.$$

These values are taken from the Masterson *et al.* paper [3] and are in agreement with todays data [22]. The leading correction in Eq. (9) comes from the factor C_M and later on when looking for CSV-effects, we will take into account this factor only.

Notice that in the approximation considered above, the quantity A_π, according to Eq. (8), does not depend on scattering angle θ. This is the consequence of the simplification we used. Namely, we took into account the impulse approximation with the πN scattering in the P_{33} wave. As was demonstrated in [3], the inclusion of others S- and P-waves does not change the picture dramatically but leads to a smooth dependence of A_π versus scattering angle θ. (Note, the deviation from calculated constant value much is smaller than the experimental data.) Nevertheless, as was shown in [3], the inclusion of the CSV effect in the form (8) already raises the possibility of describing the observed CSV on the deuteron at 143 MeV for scattering angles $\theta \le 80°$.

3. Double-Scattering Approximation

The πd differential cross section in the approximation (6) has a minimum at the scattering angle around 90°, where the non-spin-flip amplitude vanishes. For this reason, the contribution from the double-scattering term may be essential in this region of scattering angles. There are three diagrams for the double-scattering process which are depicted in Fig. 3. The sum of these amplitudes is proportional to the combination

$$\frac{1}{3}[f_{33}(k)]^2 + \frac{1}{3}[f_{33}(k)]^2 - \frac{2}{9}[f_{33}(k)]^2, \tag{10}$$

where the last term comes from the diagram with the virtual charge-exchange (Fig. 3c). To estimate the contribution of diagrams of Fig. 3, let us use the so-called fixed-centers approximation. This method for πd scattering was first used by Brueckner [23]. Its accuracy was later estimated by Kolybasov and Kudryavtsev [24]. The expression of the double-scattering diagrams without elementary πN spin-orbit forces in this fixed centers approximation has the form [24]:

$$\begin{aligned} f^{(2)}_{\pi d} &= \frac{4}{3} f_{33}(k)\ 2\ F_2(\theta,k) \\ &= \frac{4}{3} f_{33}(k)\ 2\ (1-\frac{1}{3}) f_{33}(k)\ \hat{\vec{k}}_i\ \hat{\vec{k}}'_j \\ &\int \Psi_D^2(r) e^{i(\frac{\vec{k}+\vec{k}'}{2})\cdot\vec{r}}(h_1(r)\ \hat{\vec{r}}_i\ \hat{\vec{r}}_j] + h_2(r)\delta_{ij})\ d\vec{r}, \end{aligned} \tag{11}$$

where the functions $h_1(r)$ and $h_2(r)$ are

$$h_1(r) = \frac{e^{ikr}}{r} - \frac{3e^{ikr}}{k^2r^3} + \frac{3}{k^2r^3} + \frac{3ie^{ikr}}{kr^2}, \tag{12}$$

$$h_2(r) = \frac{e^{ikr}}{k^2r^3} - \frac{1}{k^2r^3} - \frac{ie^{ikr}}{kr^2}. \tag{13}$$

This form of the functions $h_1(r)$ and $h_2(r)$ corresponds to a certain choice for the off-shell dependence for $f_{\pi N}$ amplitudes. For more details see [24]. In expression (11), $\hat{\vec{k}}$ and $\hat{\vec{r}}$ are the units vectors, $\hat{\vec{k}} = \vec{k}/k$, $\hat{\vec{r}} = \vec{r}/r$, and $\hat{k}_i$ is the i-component of this vector.

The sum of the single- and double-scattering diagrams in this approximation [1] is

$$f_{\pi d}^{(1+2)} = \frac{4}{3} f_{33}(k)\ 2\ [F_D(\theta)cos\theta + ReF_2(\theta) + i\ ImF_2(\theta)]. \tag{14}$$

The functions $F_D(\theta)cos\theta$, $ReF_2(\theta)$, and $ImF_2(\theta)$ are shown in Figure 4. We see from this Figure that the amplitude of double-scattering is strongly suppressed at forward angles versus single-scattering. But at larger than 90°-angles, the contributions of single- and double-scattering are comparable. Clearly, the inclusion of the interference effects at this angular range will be essential.

4. Spin-Flip Amplitude

Now, we take into account both the non-spin part and spin-flip parts of the elementary πN-amplitude (3). As in our previous discussion, we will take into account the single- and double-scattering terms without any recoil effects (i. e. in the fixed-center approximation). In this case, the πd-amplitude is a matrix in spin space, see example Eq. (5). After averaging over initial and summation over final polarization, we can write the final result for the cross section as the sum of three terms:

$$\sigma(\theta) = \sigma_{11}(\theta) + \sigma_{12}(\theta) + \sigma_{22}(\theta), \tag{15}$$

where $\sigma_{11}(\theta)$ is the contribution from the single-scattering, $\sigma_{22}(\theta)$ is the contribution from double-scattering, and $\sigma_{12}(\theta)$ is the single-double interference term. The expressions for these cross sections are given below:

$$\sigma_{11}(\theta) = \frac{2}{3} \mid A_1 \mid^2 (1 + 5z^2), \tag{16}$$

$$\sigma_{12}(\theta) = \frac{2}{3} Re[A_1^* A_2[(4 + 11z + 9z^2)J_1 + (8 + 20z^2)J_2]], \tag{17}$$

[1]We omit temperarily the spin-flip amplitudes taking into account only the non-spin-flip amplitudes. The inclusion of spin-flip will be done later.

$$\sigma_{22}(\theta) = \frac{1}{3} \mid A_2 \mid^2 [\frac{1}{4}(75 + 90z + 27z^2) \mid J_1 \mid^2$$
$$+(16 + 25z + 15z^2)(J_1 J_2^* + J_1^* J_2) + (34 + 34z^2) \mid J_2 \mid^2], \quad (18)$$

where $z = \cos\theta$.

The values A_1 and A_2 for the case of $\pi^+ d$ scattering are [2]

$$A_1 = \frac{2(m+\omega)}{2m+\omega}(f_1 + \frac{1}{3}f_2)F_D(\theta),$$
$$A_2 = \frac{8\pi(m+\omega)^2}{m(2m+\omega)}\frac{2}{3}f_2(f_1 - \frac{1}{3}f_2), \quad (19)$$

where $f_i = \frac{1}{2k_{cm}}\frac{\Gamma}{w_i - w - i\Gamma/2}$. If mass splitting is absent, then Eqs. (19) reduces to

$$A_1^{(0)} = \frac{2(m+\omega)}{2m+\omega}\frac{4}{3}f_0 F_D(\theta),$$
$$A_2^{(0)} = \frac{8\pi(m+\omega)^2}{m(2m+\omega)}\frac{4}{9}f_0^2, \quad (20)$$

where $f_0 = \frac{1}{2k_{cm}}\frac{\Gamma_0}{w_0 - w - \frac{i}{2}\Gamma_0}$.

With the view that the leading CSV-correction comes from the mass splitting and this splitting is small, it would be interesting to consider the formula for the cross section linearized in δm. In this case, the expression for asymmetry has the form:

$$A_\pi = -\frac{C_M}{2\sigma^{(0)}\Gamma}[3(B_0 + B_0^*)\ [\frac{1}{2}\sigma_{11}^{(0)}(\theta) + \sigma_{22}^{(0)}(\theta)]$$
$$+2Re[A_1^* A_2\ (B_0 + \frac{1}{2}B_0^*)\ [(4 + 11z + 9z^2)J_1 + (8 + 20z^2)J_2]]], \quad (21)$$

and correspondingly ratio $r = 1 + 2\ A_\pi$. Here: $B_0 = \frac{\Gamma_0/2}{w_0 - w - i\Gamma_0/2}$; the values $\sigma^{(0)}$, $\sigma_{11}^{(0)}$, and $\sigma_{22}^{(0)}$ are defined by Eqs. (15), (16), and (18), respectively, after the substitutions $A_1 \to A_1^{(0)}$ and $A_2 \to A_2^{(0)}$ from Eqs. (20). The quantities J_1 and J_2 are complex functions which depend on k and θ. They depend on the deuteron wave function as well, see Appendix.

Hence all the CSV-corrections depend on the same linear combination of masses, as in the single-scattering term, i. e. on parameter $C_M \simeq 4.6\ MeV$. Note that the inclusion of the double-scattering introduces no new parameters, i. e. the effect is still primarily dominated by C_M.

[2] Here and below we use relativistic normalization for the amplitudes.

5. Coulomb Interaction

Now, we consider the fact that the charged pions interact with the deuteron by the Coulomb force. The elementary πN-amplitude, which corresponds to the interaction of a pion with a proton via γ-exchange, is drawn in Figure 5. In terms of bi-spinors, the expression for this diagram is

$$M_{\pi p}^{(\gamma)} = \frac{4\pi e^2}{t} \bar{u}_2 (k_1 + k_2)_\mu \gamma^\mu u_1 .$$

Neglecting the magnetic interaction and adding the Coulomb phase, we finally get for the Coulomb amplitude

$$f^\gamma = \frac{M_{\pi p}^{(\gamma)}}{8\pi(m+\omega)} = -\frac{e^2}{2k_{cm}^2 \sin^2(\frac{\theta}{2})} \frac{\omega m}{(m+\omega)} e^{\left[-\frac{2ie^2}{k_{cm}} \frac{\omega m}{(m+\omega)} \ln\left(\sin\frac{\theta}{2}\right)\right]}, \tag{22}$$

where $e^2 = \frac{1}{137}$. In that follows, we add the Coulomb interaction in the single-scattering term only (in non-spin-flip part of the triangle diagram). Let us introduce in addition to the expressions A_i (19) the expression A_C

$$A_C = \frac{2(m+\omega)}{2m+\omega} (f_1 + f^\gamma + \frac{1}{3} f_2) F_D(\theta). \tag{23}$$

In terms of these amplitudes, the final expressions for cross sections $\sigma_{11}(\theta)$ and $\sigma_{12}(\theta)$ have the form:

$$\sigma_{11}(\theta) = \frac{2}{3} [6z^2 \mid A_C \mid^2 + (1 - z^2) \mid A_1 \mid^2], \tag{24}$$

$$\sigma_{12}(\theta) = \frac{2}{3} Re[A_C^* A_2 \ [(11z + 13z^2) J_1 + 28z^2 \ J_2] + A_1^* A_2 [(4 - 4z^2) J_1 + (8 - 8z^2) J_2]]. \tag{25}$$

As the Coulomb interaction was included only in the single-scattering term, the expression for $\sigma_{22}(\theta)$ remains the same as in Eq. (18). The curves for asymmetry A_π with the Coulomb interaction taken into account are given in Figures 6. If we consider the $\pi^- d$ scattering instead of $\pi^+ d$, we interchange the following terms in the expressions (22) and (23): $f_1 \to f_4$, $f_2 \to f_3$, and $f^\gamma \to -f^\gamma$. From Fig. 6, we see that single-scattering does not depend on the scattering angle but a change of sign of the asymmetry does occur between 180 and 220 MeV according to the expression, given by Eq. (9).

IV. CONCLUSION AND FUTURE PROSPECTS

In making comparisons of the experimental data for asymmetries (Fig. 1) and the corresponding theoretical curves (Figs. 6), we conclude that CSV-effects due to the double-scattering terms are indeed very small and within uncertainties of experimental data. Our approach gives indications of some enchancement of A_π in the region of angles around 90 degrees. For example, at $T_\pi = 180$ MeV (in a range of maximum effect of the Delta) there is evidence for the growth of A_π from $A_\pi = 0.002$ at $\theta = 50^\circ$ to $A_\pi = 0.015$ at $\theta = 85^\circ$ (We can expect some enhancement at 85° due to the behaviour of $F_D(\theta)cos\theta$, $ReF_2(\theta)$, and $ImF_2(\theta)$ shown in Figure 4.) But the growth of A_π is not large. The energy behaviour of A_π at 85° is shown on Fig. 7. At the same time, experimental errors for asymmetry in this region of angles are the order of one percent. The same is true for other energies. We conclude that to confirm these theoretical predictions for the asymmetry on the deuteron, one needs to have data that are approximately $2-3$ times better in precision than currently available.

The situation may be quite different in the $^3He/^3H$-case. There are two arguments as to why one may expect the CSV-effect to be larger for these nuclei:

- The enchancement of effect in $^3He/^3H$ case in comparison to deuteron may take place because of a smaller role of the spin-flip terms in the single-scattering approximation. In this approximation for the deuteron case, the ratio of non-spin-flip to spin-flip terms in the cross section is 6:1. This ratio is quite a bit larger for the $^3He/^3H$-case. So, the role of double-scattering terms in the region of angles around 90 degrees may be much more pronounced for these nuclei.

- The number of double-scattering diagrams is also increasing due to increase of possible number of rescattering combinations. This further enhances the role of double-scattering terms in comparison to the deuteron case.

The role of Fermi motion has not been discussed. This is primarily because the main aim of this work has been to investigate processes which could possibly reproduce the observed structure in πd asymmetries. Fermi motion is expected to broaden the "signal" but not lead to the sought-after structures. Moreover, in the case of the deuteron, where the asymmetry signal, both observed and calculated, is small, it is presumebly premature to discuss corrections before the magnitude of the effect is reasonably understood.

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V. APPENDIX

Here we give the expressions for the integrals J_1 and J_2.

$$J_1 = \frac{1}{4}\int dr\ r^2\psi^2(r)[(3E_2 - E_0)h_1(r)],$$
$$J_2 = \frac{1}{4}\int dr\ r^2\psi^2(r)[(E_0 - E_2)h_1(r) + 2E_0h_2(r)]. \tag{26}$$

Here $E_n = \int\limits_{-1}^{+1} e^{i\kappa r z} z^n dz$, $\kappa = k\cos(\frac{\theta}{2}) = k\sqrt{(1+z)/2}$ and functions $h_1(r)$ and $h_2(r)$ were given in the main text, see Eqs. (12) and (13).

Let us calculate the integral J_1. For this purpose, it is suitable to use the following representation for underintegral function:

$$(3E_2 - E_0)h_1(r) = \sum_{m=1}^{16} a_m \frac{e^{ib_m r}}{r^{n_m}}. \tag{27}$$

Here $n_m = 2,\ 3,\ 4,\ 5,\ 6,\ 4,\ 5,$ and 6 for $m = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,$ and 8, respectively; $n_m = n_{m-8}$ for $9 \le m \le 16$, and

$$\begin{aligned}
a_1 &= -2ik^{-1}x^{-1},\\
a_2 &= 6k^{-2}x^{-1}(1 + x^{-1}),\\
a_3 &= 6ik^{-3}x^{-1}(1 + 3x^{-1} + x^{-2}),\\
a_4 &= -18k^{-4}x^{-2}(1 + x^{-1}),\\
a_5 &= -18ik^{-5}x^{-3},\\
a_6 &= -6ik^{-3}x^{-1},\\
a_7 &= 18k^{-4}x^{-2},\\
a_8 &= 18ik^{-5}x^{-3},\\
b_1 &= b_2 = b_3 = b_4 = b_5 = (1 + x)\ k,\\
b_6 &= b_7 = b_8 = x\ k,
\end{aligned} \tag{28}$$

where $x = \cos(\frac{\theta}{2})$. These Eqs. (28) after the replacement $x \to -x$ define the values a_m and b_m for $9 \le m \le 16$ as $a_m = a_{m-8}$ and $b_m = b_{m-8}$.

In calculations, we use a realistic deuteron wave function (in S-wave approximation) of the Bonn potential [25], parametrized as $\psi(r) = \sum_i c_i \frac{e^{-\alpha_i r}}{r}$, where $\alpha_i > 0$. With this form of $\psi(r)$, we get

$$J_1 = \frac{1}{4}\int \sum_{ijm} c_i c_j a_m e^{(ib_m - \alpha_i - \alpha_j)r} \frac{dr}{r^{n_m}}. \tag{29}$$

To take this integral, one may use a general relation

$$\int_0^\infty \sum_i c_i e^{a_i x} \frac{dx}{x^{n_i}} = \sum_i c_i \frac{a_i^{n_i - 1}}{(n_i - 1)!}[S_{n_i - 1} - \ln a_i], \tag{30}$$

where $S_n = \sum_{k=1}^{n} \frac{1}{k}$ and $S_0 = 0$. The formula (27) is derived for the case $n_i \geq 1$ and is valid if this integral converges (i. e. $Re\, a_i < 0$ and the underintegral function is finite at $x \to 0$). These conditions are satisfied for the integral (26), and we finally get:

$$J_1 = \frac{1}{4} \sum_{ijm} c_i c_j a_m \frac{(ib_m - \alpha_i - \alpha_j)^{n_m - 1}}{(n_m - 1)!} \; (S_{n_m - 1} - \ln \sqrt{(\alpha_i + \alpha_j)^2 + b_m^2} + i \; atan \frac{b_m}{\alpha_i + \alpha_j}), \quad (31)$$

To obtain the expression for J_2, one may use the analogous representation

$$(E_0 - E_2) h_1(r) + 2E_0 h_2(r) = \sum_{m=1}^{14} a_m \frac{e^{ib_m r}}{r^{n_m}}, \quad (32)$$

Here: $n_m = 3,\ 4,\ 5,\ 6,\ 4,\ 5,$ and 6 for $m = 1,\ 2,\ 3,\ 4,\ 5,\ 6,$ and 7, respectively; $n_m = n_{m-7}$ for $8 \leq m \leq 14$ and

$$\begin{aligned} a_1 &= -2k^{-2}x^{-1}(1 + x^{-1}), \\ a_2 &= -2ik^{-3}x^{-1}(1 + 3x^{-1} + x^{-2}), \\ a_3 &= 6k^{-4}x^{-2}(1 + x^{-1}), \\ a_4 &= 6ik^{-5}x^{-3}, \\ a_5 &= 2ik^{-3}x^{-1}, \\ a_6 &= -6k^{-4}x^{-2}, \\ a_7 &= -6ik^{-5}x^{-3}, \\ b_1 &= b_2 = b_3 = b_4 = (1 + x)\ k, \\ b_5 &= b_6 = b_7 = x\ k, \end{aligned} \quad (33)$$

where $x = \cos(\frac{\theta}{2})$. These Eqs. (33) after the replacement $x \to -x$ define the values a_m and b_m for $8 \leq m \leq 14$ as $a_m = a_{m-7}$ and $b_m = b_{m-7}$. Thus, for the integral J_2 we get the similar Eqs. (28) in which the values n_m, a_m, and b_m are defined by Eqs. (29) and (30).

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Figure captions

Figure 1. Asymmetry A_π at different energies. (a) 30 MeV, (b) 50 MeV, (c) 65 MeV, (d) 143 MeV, (e) 180 MeV, (f) 220 MeV, (g) 256 MeV, and (h) 417 MeV. Experimental data are from [7] (open circles), [6] (open triangles), [3] (filled triangles), [9] (filled circles), [2] (open diamonds), [5] (stars), [4] (filled squares), and [8] (filled diamonds). Individual πd elastic scattering PWA (combined PWA of pp and πd elastic scattering with $\pi d \rightarrow pp$) result for asymmetry is shown by dash-dotted (dashed) curve [21]. The asymmetry data were not use in PWAs.

Figure 2. Single-scattering amplitudes for $\pi^+ d$ on the proton (a) and the neutron (b).

Figure 3. Double-scattering amplitudes for $\pi^+ d$: elastic (a) and (b), and with virtual charge-exchange (c).

Figure 4. Amplitudes for πd scattering without spin-flip at 140 MeV. Solid curve gives $F_D(\theta) cos\theta$. The real (imaginary) parts of amplitude $F_2(\theta)$ is plotted with dash-dotted (dashed) lines.

Figure 5. Feynman diagrams for the Coulomb πp and πd amplitudes.

Figure 6. Asymmetry for πd scattering with the Coulomb interaction taken into account. (a) 143 MeV, (b) 180 MeV, (c) 220 MeV, and (d) 256 MeV. Experimental data are from [2] $-$ [4], [6] $-$ [9]. Notation is the same as in Fig. 1. Solid curves give the total amplitude. Single (and double) scattering without Coulomb corrections is shown by dashed (dash-dotted)curves.

Figure 7. 85° energy dependence of A_π for πd scattering with the Coulomb interaction taken into account. Notation is the same as is in Fig. 6.